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A REVIEW OF THE STATISTICAL PROPERTIES OF THE OCEAN SURFACE. (U)
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NAVAL UNDERWATER SYSTEMS CENTER
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A REVIEW OF THE STATISTICAL PROPERTIES OF THE OCEAN SURFACE

by

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INTRODUCTION

The physical principles of the deterministic properties of surface waves have been studied for over a century. The work by Lamb[1] is one of the classic studies in this field. The consideration of the statistical properties of the ocean surface was first started about 20 years ago. One of the first publications on this subject was by Pierson, Neumann and James[2]. This manual is informative and readable. Two excellent surveys of ocean statistics have been given by Kinsman[3] and Cartwright[4]. One of the original and much quoted work on surface statistics was done under project SWOP in 1960[5]. A conference in ocean wave spectra[6] contains the work of many scientists in the field.

The purpose of this study ~~is to~~ review the work on ocean surface statistics. and In this review some new approaches are considered. A brief discussion of some similarities between ocean surface statistics and underwater sound noise fields is included. It is hoped that this review will be helpful as an introduction to the field.

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ADMINISTRATIVE INFORMATION

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1

Single Frequency Plane Waves

Let us first consider a single frequency plane wave moving in the direction θ_n. Let the amplitude of the wave be a_n and the phase ϵ_n. Let $\eta(x, y, t)$ be the displacement of the water height from its equilibrium position, where x and y represent a point on the surface and t is the time. Then

$$\eta(x,y,t) = a_n \cos\left(\frac{\omega^2}{g}\cos\theta_n x + \frac{\omega^2}{g}\sin\theta_n y - \omega t - \epsilon_n\right) \qquad \#1$$

In this equation, we have assumed that the surface wave is for the deep water case and for this case the wave number $k = \omega^2/g$, where g is the gravity acceleration constant. Equation #1 defines the surface height for all points on the surface and for all time. It is a three dimensional wave, two of the dimensions are the x and y coordinates and the third dimension is the displacement of the surface in the z direction given by $\eta(x, y, t)$. Let us now assume that there are single frequency plane waves of the same frequency each travelling in a different direction. Then the displacement is

$$\eta(x,y,t) = \sum_{i=1}^{N} a_n \cos\left(\frac{\omega^2}{g}\cos\theta_n x + \frac{\omega^2}{g}\sin\theta_n y - \omega t - \epsilon_n\right) \qquad \#2$$

In equation 2, we have assumed that the principle of superposition holds, i.e. that the total displacement is equal to the linear sum of the individual displacements. In general this is not true. However, for small displacements, the approximation of assuming linear superposition is good. In addition, it makes much of the theory considerably simpler and many experimental results may be predicted from the linear superposition model. A given plane wave travelling in the θ_n direction, has an amplitude a_n and phase ϵ_n. Up to this point the description has been deterministic. We now introduce the random model by considering an ensemble of surfaces similar to the deterministic surface. Each surface consists of N single frequency plane waves. The n^{th} plane wave is travelling in the θ_n direction with an amplitude a_n. For the n^{th} plane wave in a given surface, the phase ϵ_n is selected from a population with phase between 0 and 2π, all phases being equally likely. In this model, each surface is identical except for the phase ϵ_n. If we obtain the probability distribution of the phase over an ensemble of surfaces, then the probability density is shown in Fig. 1.

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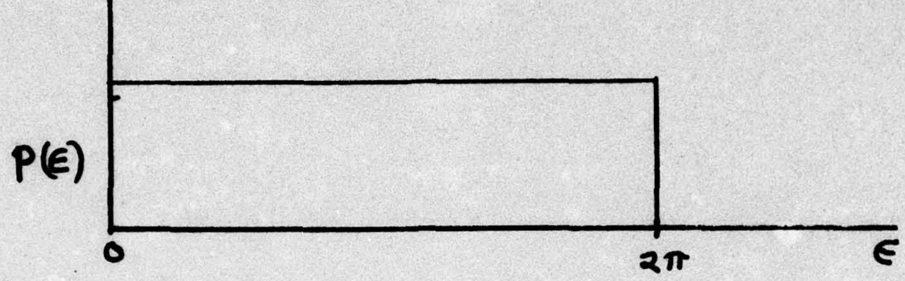

Fig. 1.

Because of the random phase condition, each surface displacement configuration is different than the others. It should be noted that the model could be constructed by having random amplitudes a_n in addition to random phase, or random amplitudes alone.

Let us now obtains ome of the statistical properties of the ensemble. However, before we do this, let us review some definitions that will be used. A stationary random process is one whose statistical properties do not change with time. An ergodic process is a stationary process such that with probability one, the time average of any single function of the ensemble is equal to the ensemble average. It should be noted that a process to be ergodic is that each sample of the ensemble go through all possible points of the set. The model as described above is stationary but not ergodic. For example, a given surface is a superposition of sinusoidal plane waves of a given frequency, so that it is a sinusoidally oscillating surface in time of the same frequency as its components. The configuration that the individual surface can take is quite broad, but the frequency of each individual spatial point is still sinusoidal. However, the amplitude of the oscillations at the point is fixed and will thus not necessarily reach all possible values. There are several ways to circumvent this problem. However, the method of single frequencies is very helpful intuitively and since the process will become ergodic when continuum of frequencies is introduced, we will use it here.

We will now average over the ensemble of systems. In order to hold the amount of symbols to a minimum, we will consider a two dimensional wave surface. The extension to a three dimensional surface is obvious, once the two dimensional surface is understood. For a two dimensional surface,

$$\eta(x,t) = \sum_{n=1}^{N} a_n \cos\left(\frac{\omega^2}{g}x - \omega t - \epsilon_n\right) \qquad \#3$$

We now assume that the process is stationary in the wide sense in both space and time. That is the second order properties, such as correlation, depend only on the difference in x and the difference in t and not on where we begin. We let $v = x_2 - x_1$, $\tau = t_2 - t_1$

then $\langle \eta(x,t)\eta(x-v,t-\tau)\rangle$

$$= \left\langle \sum\sum a_m a_n \cos\left(\frac{\omega^2}{g}x - \omega t - \epsilon_m\right)\cos\left(\frac{\omega^2}{g}(x-v) - \omega(t-\tau) - \epsilon_n\right)\right\rangle$$

The symbol $\langle\ \rangle$ denotes the expected value over the ensemble. We now use the trigonometric identity that relates the product of cosines to the sum and differences of the arguments of cosines. Then

$$\langle \eta(x,t)\eta(x-v,t-\tau)\rangle = \tfrac{1}{2}\left\langle \sum\sum a_m a_n \cos\left[\frac{\omega^2}{g}x + \frac{\omega^2}{g}(x-v) - \omega\tau - \omega(t-\tau) - \epsilon_m - \epsilon_n\right] + \cos\left[\frac{\omega^2}{g}v - \omega\tau - (\epsilon_m - \epsilon_n)\right]\right\rangle$$

We note that the first term on the right hand side is always zero when averaged over ϵ. That is, the phase difference remains when $m \neq n$ and when $m = n$, $\langle \cos(a+\epsilon)\rangle_\epsilon = 0$

When $m \neq n$ the second term is also zero, but when the phases in the second term cancel, we have

$$\langle \eta(x,t)\eta(x-v,t-\tau)\rangle = \sum_{n=1}^{N} a_n^2 \cos\left(\frac{\omega^2}{g}v - \omega\tau\right)$$

The mean square surface height σ^2 is defined as

$$\sigma^2 = \langle \eta^2(x,y,t)\rangle$$

We now define the spatial temporal correlation of the surface height as

$$\rho(v,V,\tau) = \frac{\langle \eta(x,y,t)\eta(x-v,y-V,t-\tau)\rangle}{\sigma^2}$$

Then using similar arguments as in the two dimensional case

$$\sigma^2\rho(v,V,\tau) = \sum_{n=1}^{N} a_n^2 \cos\left(\frac{\omega^2}{g}\cos\theta_n v + \frac{\omega^2}{g}\sin\theta_n V - \omega\tau\right) \qquad \#4$$

Equation 4, related the spatial-temporal correlation of the ocean surface, for single frequency waves, to the value of a_n^2 which is proportional to the power of a given component wave in the θ_n direction. We can now extend this approach to include other frequencies. Then a is a function of direction θ and frequency ω. Finally, we can let this double sum increase and let the increments between values decrease so that the double sum increase and let the increments between values decrease, so that the double sum approaches the double integral. An approach of this type has been given by Pierson and is reported by Kinsman[3]. Kinsman's equation in our notation is

$$\sigma^2\rho(v,V,\tau) = \frac{1}{2}\int_{-\pi}^{\pi} d\theta \int_0^\infty d\omega\, A^2(\omega,\theta)\cos\left(\frac{\omega^2}{g}v\cos\theta + \frac{\omega^2}{g}V\sin\theta - \omega\tau\right) \qquad \#5$$

$A^2(\omega,\theta)$ is called the directional wave spectrum. $A^2(\omega,\theta)\,d\omega\,d\theta$ is the relative amount of power of the component surface waves at (ω,θ) in the range $d\omega\,d\theta$. A three dimensional plot of $A^2(\omega,\theta)$ is given by Kinsman[3] in Fig. 8. 3-1. This equation 5 represents the relation between the spatial-temporal surface correlation and the directional wave spectrum.

In this section, we have gone to great lengths to treat the single frequency case and to obtain the relationship given by equation 4. We have indicated that the same type of reasoning could be used to obtain equation 5.

Another form of equation 5 that is useful is to express the right hand side in terms of wave numbers rather than ω and θ.

Let

$$k_x = \frac{\omega^2}{g}\cos\theta$$

$$k_y = \frac{\omega^2}{g}\sin\theta$$

Then

$$\hat{A}^2(k_x,k_y) = A^2(\omega,\theta)\, J\left(\frac{\omega,\theta}{k_x,k_y}\right)$$

When J is the Jacobian and

$$J\left(\frac{\omega,\theta}{k_x,k_y}\right) = \begin{vmatrix} \frac{\partial\omega}{\partial k_x} & \frac{\partial\omega}{\partial k_y} \\ \frac{\partial\theta}{\partial k_x} & \frac{\partial\theta}{\partial k_y} \end{vmatrix}$$

$$= \frac{\partial\omega}{\partial k_x}\frac{\partial\theta}{\partial k_y} - \frac{\partial\omega}{\partial k_y}\frac{\partial\theta}{\partial k_x}$$

If we use these transformations, we obtain the following two equations. The first relation is

$$\hat{A}^2(k_x,k_y) = \frac{\sqrt{g}}{2}\,\frac{A^2\left(\sqrt{g}\,(k_x^2+k_y^2)^{1/4},\ \tan^{-1}\left(\frac{k_y}{k_x}\right)\right)}{(k_x^2+k_y^2)^{3/4}}$$

The second equation is

$$\sigma^2\rho(u,v,\tau) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \hat{A}^2(k_x,k_y)\cos\left(k_x u + k_y v - \sqrt{g}\,(k_x^2+k_y^2)^{1/4}\tau\right) \qquad \#6$$

Properties at a Single Point

is very difficult to obtain experimentally. The method of attack as explained in the literature, is to consider various simplifications of this quantity. That is, various parameters are held fixed during a given measurement.

The simplest quantity to measure is the time variation of the surface at a given point. We use an instrument to measure the height versus time at a given point of the ocean surface. A capacitance wave pole is an example of this type of instrument. We then obtain a time history as shown in Fig. 2.

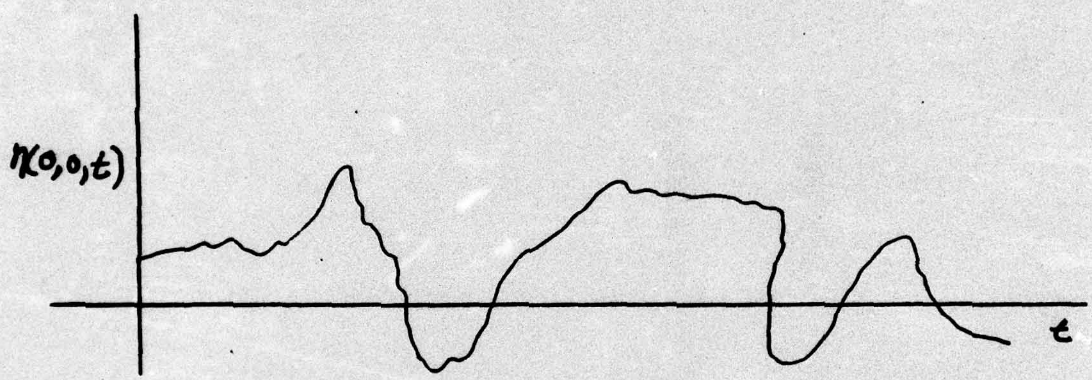

Fig. 2.

From this time history, we obtain the autocorrelation as

$$\sigma^2 \rho(0,0,\tau) = \langle \eta(0,0,t)\,\eta(0,0,t-\tau) \rangle$$

The power spectrum at this point is the Fourier transform of the autocorrelation function. Thus

$$\Phi(f) = \int_{-\infty}^{\infty} d\tau\, \rho(0,0,\tau) \exp(-j2\pi f \tau)$$

A great deal of the work in ocean statistics and the literature are concerned with the quantity $\Phi(f)$. Returning to equation 5, we have

$$\sigma^2 \rho(0,0,\tau) = \frac{1}{2}\int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} d\omega\, A^2(\omega,\theta) \cos(\omega\tau)$$

Then

$$\Phi(f) = \int_{-\infty}^{\infty} d\tau \exp(-j2\pi f\tau)\, \sigma^2 \rho(0,0,\tau)$$
$$= \pi \int_{-\pi}^{\pi} d\theta\, A^2(2\pi|f|,\theta) = \pi A^2(\omega), \quad \omega \geq 0$$

Note that $\Phi(f)$ is a double sided spectrum and $A^2(\omega)$ is single sided. Now consider the right hand side of the last equation, i.e.

$$A^2(\omega) = \int_{-\pi}^{\pi} d\theta\, A^2(\omega,\theta)$$

$A^2(\omega, \theta)$ is the directional wave spectrum. In the equation, we integrate over all θ. Thus all knowledge of the direction of the waves is lost, if we consider only the quantity $A^2(\omega)$. For the Neumann-Pierson spectrum

$$A^2(\omega, \theta) = \frac{C}{\omega^6} \exp\left(-\frac{2g^2}{\omega^2 U^2}\right) \cos^2\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \qquad \#7$$
$$\omega_1 \leq \omega < \infty.$$

Then

$$A^2(\omega) = C\,\frac{\pi}{2}\,\frac{1}{\omega^6} \exp\left(-\frac{2g^2}{\omega^2 U^2}\right), \quad \omega_1 \leq \omega < \infty \qquad \#8$$

= 0, otherwise

where g = 980 cm/sec.^{2}
U = wind speed in cm/sec.
C = 3.05×10^4 cm^2/sec.^{5}
W_1 = is a cutoff frequency
$\omega_1 = 0$ for a fully aroused sea. If we plot $A^2(\omega)$ Vs ω we obtain a curve as shown in Fig. 3.

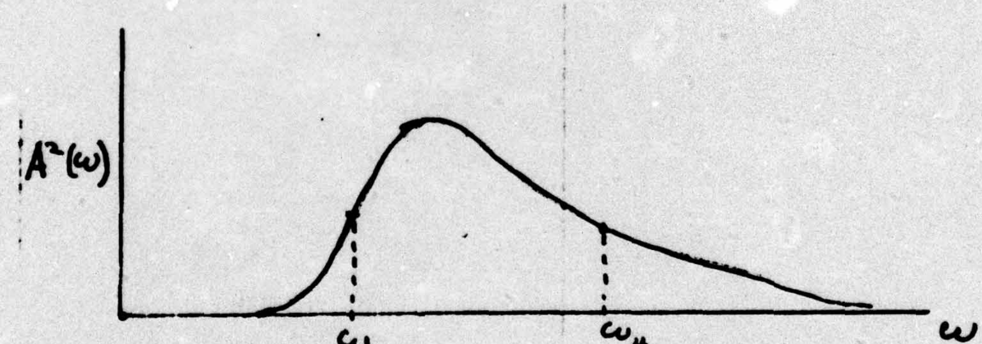

Fig. 3.

For a higher value of wind speed U, the peak of this curve shifts to a lower value of w and becomes narrower. If we define Q as

$$Q = f_p/f_H - f_L = \frac{\omega_p}{\omega_H - \omega_L}$$

where fp is the peak frequency and f_L and f_H are the frequencies at the half power points, then $Q = 1.4$ for all values of U. Thus the Neumann-Pierson spectrum represents a wide

band process. The integration over ω in equation 8, gives the total power at a point. Thus

$$\text{Power} = \int_0^\infty \hat{A}^2(\omega)\, d\omega = \int_0^\infty C\,\frac{\pi}{2}\,\frac{1}{\omega^6}\,\exp\left(-\frac{2g^2}{\omega^2 U^2}\right) d\omega$$

Properties for a Given Time

Let us now return to equation #6 and consider the spatial correlation of the surface height at an instant. Then

$$\sigma^2 \rho(u,v,0) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \hat{A}^2(k_x,k_y)\cos(k_x u + k_y v)$$

We can now express $\hat{A}^2(k_x, k_y)$ in terms of its even and odd components. Thus

$$\hat{A}_E^2(k_x,k_y) = \frac{1}{2}\left[\hat{A}^2(k_x,k_y) + \hat{A}^2(-k_x,-k_y)\right]$$

$$\hat{A}_O^2(k_x,k_y) = \frac{1}{2}\left[\hat{A}^2(k_x,k_y) - \hat{A}^2(-k_x,-k_y)\right]$$

Thus

$$\hat{A}_E^2(k_x,k_y) = \frac{1}{4\pi^2}\iint \sigma^2 \rho(u,v,0)\cos(k_x u + k_y v)\, du\, dv$$

and #9

$$\hat{A}_O^2(k_x,k_y) = \frac{1}{4\pi^2}\iint \sigma^2 \rho\left(u,v,\frac{\pi}{2\omega}\right)\sin(k_x u + k_y v)\, du\, dv,$$

$$\omega = \omega(u,v)$$

Now the instantaneous height of the surface may be obtained by photogrammetric techniques as shown by Pierson[5]. From the instantaneous heights, the correlation $\rho(u,v,0)$ may be computed. If we then use equation 9, we may numerically integrate the right hand sides of this equation and thus obtain $\hat{A}_E^2(k_x,k_y)$ and $\hat{A}_O^2(k_x,k_y)$. Finally $\hat{A}^2(k_x,k_y)$ is obtained from its even and odd components. It should be noted that $\rho(u,v,0)$ may be obtained by stereo methods. However, $\rho\left(u,v,\frac{\pi}{2\omega}\right)$ requires photographs at different times. This value is difficult to obtain in practice. The difficulty is overcome, if it is assumed that the directional wave spectrum occurs only in two

adjacent quadrants about the wind axis. That is, the surface wave directions are always within $\pm\frac{\pi}{2}$ of the wind axis. Methods of this type have been used in project SWOP (Stereo Wave Observation Project)[5].

For analytic purposes, the problem is simplified if it is assumed that there is 180° symmetry in the directional wave spectrum, i.e.

$$\hat{A}^2(-k_x,-k_y) = \hat{A}^2(k_x,k_y)$$

$$\sigma^2\rho(u,v,0) = \iint_{-\infty}^{\infty} dk_x\,dk_y\,\hat{A}^2(k_x,k_y)\exp\left[j(k_x u + k_y v)\right] \qquad \#10$$

From equation 10, it is seen that the two dimensional Fourier transform of $\hat{A}^2(k_x,k_y)$ is $\rho(u,v,0)$. Thus $\hat{A}^2(k_x,k_y)$ is the inverse two dimensional Fourier transform of $\rho(u,v,0)$ That is,

$$\hat{A}^2(k_x,k_y) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} du\,dv\,\sigma^2\rho(u,v,0)\exp\left[j(k_x u + k_y v)\right] \qquad \#11$$

We now make the additional assumption that the surface is isotropic. An assumption of this type was made by W. Marsh[7] and also by Nuttall and Cron[8]. By an isotropic surface, we mean that the directional wave spectrum is the same in all directions.

Let $r = \sqrt{u^2+v^2}$, $\alpha = \sqrt{k_x^2+k_y^2}$

Then $\rho(u,v,0) = \rho_R(r)$

Let $k_x = \alpha\cos\phi$, $k_y = \alpha\sin\phi$ and then $u = r\cos\theta$, $v = r\sin\theta$

$$\hat{A}_R^2(\alpha,\phi) = \frac{1}{4\pi^2}\int_0^{\infty}\int_0^{2\pi}\sigma^2\rho_R(r)\exp\left[-j(\alpha r\cos\theta\cos\phi + \alpha r\sin\theta\sin\phi)\right]r\,dr\,d\theta$$

$$\hat{A}_R^2(\alpha,\phi) = \frac{1}{4\pi^2}\int_0^{\infty} dr\,\rho_R(r)\,r\int_0^{2\pi} d\theta\,\exp\left[-j\alpha r\cos(\theta-\phi)\right]$$

If we use the relation for the Bessel function of the first kind, i.e.

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi}\exp(jx\cos\theta)\,d\theta$$

Then
$$\hat{A}_R^2(\alpha) = \frac{1}{2\pi}\int_0^{\infty} dr\,r\,\sigma^2\rho_R(r)J_0(\alpha r) \qquad \#12$$

This is a Fourier-Bessel transform. The inverse transform is of the same form. That is

$$\sigma^2\rho_R(r) = 2\pi\int_0^{\infty} d\alpha\,\alpha\,\hat{A}_R^2(\alpha)\,J_0(\alpha r) \qquad \#13$$

In the preceding discussion we have considered the rectangular form of the directional wave spectrum, i.e. $\hat{A}^2(k_x, k_y)$. We then found the relation between the spatial correlation of the surface and the directional wave spectrum, for the isotropic case. The two quantities are Fourier-Bessel transforms of one another. The directional wave spectrum is usually expressed in the polar form. For example the Neumann-Pierson spectrum in polar form is given in equations #7 and #8. We will now derive the relation between the spatial correlation and the directional wave spectrum, expressed in the polar form, for the isotropic case. Let $\tau = 0$, in equation #5. Then

$$\sigma^2 \rho(U,V,0) = \frac{1}{2}\int_{-\pi}^{\pi} d\theta \int_0^{\infty} A^2(\omega,\theta) \cos\left(\frac{\omega^2}{g} U\cos\theta + \frac{\omega^2}{g} V \sin\theta\right)$$

For the isotropic case, $A^2(\omega,\theta) = A^2(\omega)$
Let $r = \sqrt{U^2+V^2}$, $\gamma = \tan^{-1}\left(\frac{V}{U}\right)$. Then

$$\sigma^2 \rho_R(r) = \frac{1}{2}\int_{-\pi}^{\pi} d\theta \int_0^{\infty} d\omega\, A^2(\omega) \cos\left(\frac{\omega^2}{g} r(\theta-\gamma)\right)$$

Then using the definition of the Bessel function of the first kind, we obtain

$$\sigma^2 \rho_R(r) = 2\pi \int_0^{\infty} d\omega\, A^2(\omega)\, J_0\left(\frac{\omega^2 r}{g}\right) \qquad \#14$$

Equation 14 has been used in Marsh[7] et al. The inverse Fourier-Bessel transform of $\rho_R(r)$ is

$$A^2(\omega) = \frac{\omega^3}{g^2}\frac{1}{\pi}\int_0^{\infty} dr\, r\, \sigma^2 \rho_R(r)\, J_0\left(\frac{\omega^2 r}{g}\right) \qquad \#15$$

Cron and Streit[9] have numerically evaluated equation 14, for the case of the Neumann-Pierson spectrum. This was done for various values of wind speed. Nuttall and Cron[8] have numerically evaluated equation 15 for various assumed values of spatial correlation.

Cross Spectral Density

Let us now consider two points of the surface. We let time vary and obtain $\rho(U,V,\tau)$ for these two points. The cross spectral density, denoted by $g(U,V,f)$ in this article, is

$$g(U,V,f) = \int_{-\infty}^{\infty} d\tau\, \rho(U,V,\tau) \exp(-j2\pi f \tau) \qquad \#16$$

Note that U and V are held fixed in the integration process. Barber has shown, as referenced by Cartwright[4] that a linear array of elements and their associated cross spectral densities can be used to

obtain the directional wave spectrum. For example, for two points in space, we could obtain the height versus time of each point, as given in Fig. 4.

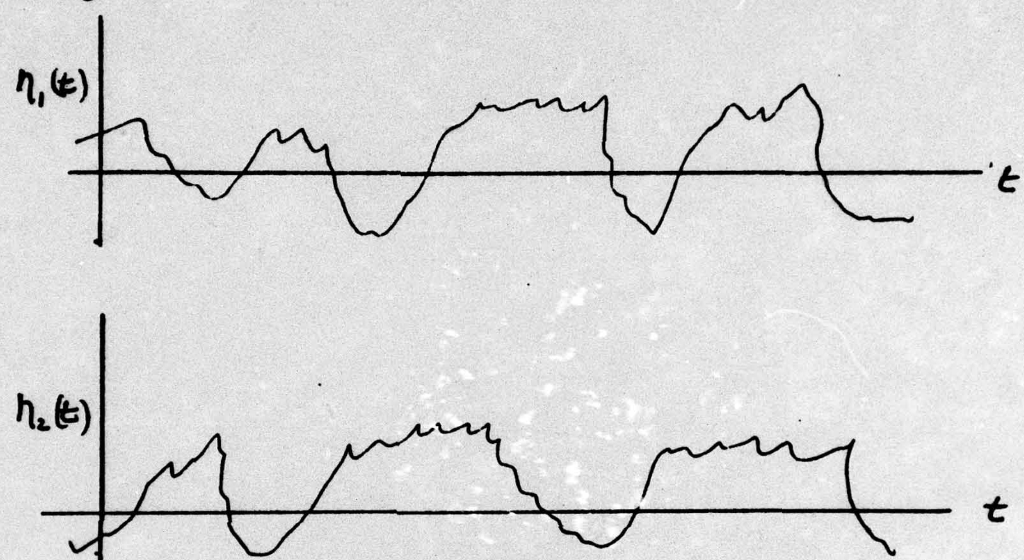

Fig. 4.

Let $\eta_1(t) = \eta(0, 0, t)$

$\eta_2(t) = \eta(x_1, y_1, t)$

Then

$$\sigma^2 \rho(u, v, \tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \eta_1(t)\,\eta_2(t-\tau)\,dt$$

Thus $\rho(u,v,\tau)$ can be obtained experimentally. The Fourier transform of $\rho(u,v,\tau)$ would yield $g(u, v, f)$. An alternate method is to FFT $\eta_1(t)$ and $\eta_2(t)$ so as to obtain $N_1(f)$ and $N_2(f)$ respectively. Then $g(u, v, f) = N_1(f)\,N_2^*(f)$ where $*$ represents the complex conjugate. For a large number of elements, this procedure is repeated for all pairs of elements.

We will now review Barber's method. Let $A^2(f)$ be the power spectrum at a point. The autocorrelation function is

$$\rho(0,0,\tau) = \langle \eta(0,0,t)\,\eta(0,0,t-\tau)\rangle = \int_{-\infty}^{\infty} A^2(f)\exp(-j2\pi f\tau)\,df$$

Thus $\rho(0,0,\tau) \longleftrightarrow A^2(f)$ where $\longleftrightarrow$ represents the Fourier Transform. We again let $A^2(f,\theta)$ represent

the directional wave spectrum for a plane wave of this power, approaching the two elements as shown in Fig. 5, we have

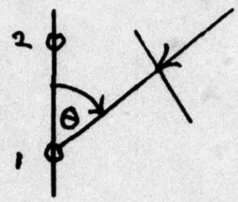

Fig. 5.

$$e(u, v, \tau + \frac{d \cos\theta}{c}) \longleftrightarrow A^2(f, \theta) \exp(j k d \cos\theta)$$

The right hand side represents the cross spectral density of a plane wave. We now consider the ocean surface to consist of plane waves from all directions. Then the total cross spectrum is

$$g(u, v, f) = \int_0^{2\pi} A^2(f, \theta) \exp(j k d \cos\theta) \, d\theta$$

Since $A^2(f, \theta)$ is periodic in θ with period 2π, we may express it as a Fourier series. Then

$$A^2(f, \theta) = \frac{A_0(k)}{2} + \sum_{i=1}^{\infty} A_i(k) \cos(i\theta) \qquad \#17$$

In this expression, we have assumed symmetry in θ. Physically, this would correspond to taking the wind direction as the axis and measuring θ from this axis. We also have the line of the linear array along the wind axis. For the general case we need two linear arrays, one along the axis and one perpendicular to the axis. Substituting equation 17 into equation 16 and using the definition of the Bessel function which is

$$J_m(x) \triangleq \frac{i^m}{2\pi} \int_0^{2\pi} \exp(j x \cos\theta) \, d\theta$$

We then obtain

$$g(u, v, f) = \pi A_0(k) J_0(kd) + 2\pi \sum_{i=1}^{\infty} j^i A_i(k) J_i(kd)$$

We can measure $g(u, v, f)$. We can compute $J_i(kd)$.
If we terminate the infinite series at the $(N-1)^{th}$ term, we have N unknowns in $A_i(k)$ where $i = 0, 1, \ldots N-1$. If we now use N receivers, we will have N equations and N unknowns. We can then solve these equations, to find the N values of $A_i(k)$. From these N values, we can compute the approximate value of $A^2(f, \theta)$ from equation 17. Thus cross spectral measurements may be used to obtain the directional wave spectrum.

A variation of the above techniques was used in the underwater sound case, by Cron, Hassell and Keltonic[10]. In that reference, various directional wave fields were considered and corresponding to each wave field, the spatial correlation of a vertical array of elements was derived analytically. Cross correlation measurements were then made on a vertical array of elements and the experimental values were compared with the values obtained from the theoretical model of various assumed directional wave spectra. The directional wave spectra that gave the best agreement was chosen as the actual directional wave spectrum. Another variation of this technique used in underwater sound is that by Von Winkle et al[11].

We will now consider the probability distribution of the ocean surface height. However, before we do, let us summarize the last section. We started by indicating how the relation between the spatial temporal correlation and the wave directional spectrum could be obtained. The spatial temporal correlation was expressed as $\rho(\upsilon, \nu, \tau)$.

We then took the special case of one point on the surface. For this case, we obtained the expression $\rho(0,0,\tau)$. This expression was related to the power spectrum. The directional properties of the waves were lost. We then specialized the general spatial temporal correlation to the case of a fixed instant of time and thus obtained $\rho(\upsilon, \nu, 0)$. This expression was related to the wave directional spectrum. We then considered two fixed points on the surface and obtained $\rho(\upsilon, \nu, \tau)$. From $\rho(\upsilon, \nu, \tau)$, the cross spectral density was obtained.

All of these properties are second order properties. For example, $\sigma^2 = \langle \eta^2(x,y,t) \rangle$ is a second order property. We have not covered higher order moments such as $\langle \eta^3(x,y,t) \rangle$ and $\langle \eta^2(x,y,t)\eta(x-\upsilon, y-\nu, t-\tau) \rangle$ which are third order moments. If the process is Gaussian, only the first and second order moments are needed. Fortunately, for the surface height distribution, the Gaussian process may be assumed for many cases.

Probability Distributions

Let us now consider the probability distribution of the surface height at a point. We will return to the principle of linear superposition. The height of the surface at a point is the sum of many sine waves travelling in different directions and with different frequencies. For a broad set of conditions, the sum of a large number of random variables results in a random variable with a Gaussian distribution, as

may be obtained from the central limit theorem. The probability distribution of the height is

$$p(h) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-h^2/2\sigma^2)$$

where h is the height and $\sigma^2 = \langle h^2 \rangle$ is the mean square height. This distribution has a bell shape and is shown in Fig. 6.

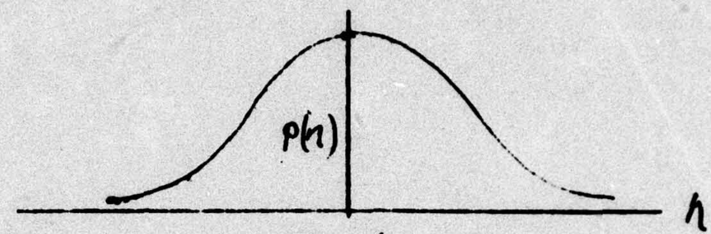

Fig. 6.

It should be noted that the principle of superposition holds for small amplitude, low frequency waves. High frequency waves result in non linear interaction terms and thus one would not expect a Gaussian process. This is borne out by experiment.

MacKay, as referenced by Kinsman[3], made an exhaustive study of the wave height distribution, by the means of bottom pressure recorders. He used the chi-square test, Kolmogorov-Smirnov test etc., to show that the Gaussian hypothesis was accepted. However, it is known that the high frequency surface waves are greatly attenuated. Thus the high frequency surface waves were not included in MacKay's experiments. Kinsman[3] obtained the probability distribution of the surface height by means of a capacitance pole. He thus included the high frequency components. A comparison between the experimental values and the Gaussian distribution showed that the two were different, but only slightly different. There is a small amount of skewness, i.e. The third moment was not zero in the experimental values, whereas the Gaussian distribution is symmetric. Since the two distributions are close, for some practical purposes, we can assume that the surface height will also be Gaussian.

If the surface is stationary in the wide sense in both space and time and if the process is Gaussian, then the directional wave spectrum or the spatial-temporal surface correlation completely specifies the statistical properties of the surface.

(cont fr p1)

SUMMARY

The author outlines

We have outlined the method for obtaining the equation showing the relation between the spatial-temporal correlation and the directional wave spectrum. Methods of measuring the spatial-temporal correlation of the surface ~~were~~ are mentioned along with the equations, and Special cases of this ~~the~~ spatial-temporal correlation ~~were~~ are considered. The most important one from the present measurement viewpoint (i. e. the simplest to measure) is the temporal correlation at a point on the surface. The Fourier Transform of this correlation leads to the power spectrum of the Neumann-Pierson type. Finally, a method of obtaining cross spectra between pairs of a linear array of elements is ~~was~~ discussed and similarities between this method and the underwater sound problem of obtaining the directionality of ambient noise is ~~was~~ mentioned.

REFERENCES

(1) Lamb, H. Hydrodynamics, Dover Publications. New York, 1945.

(2) Pierson, W., Neumann, A. and James R., Practical Methods for Observing and Forecasting Ocean Waves by Means of Wave Spectra and Statistics, Hydrographic Office Pub. No. 603, U.S. Dept. of Navy

(3) Kinsman, B., Wind Waves, Prentice Hall, 1965

(4) Cartwright, D., Article in "The Sea" Volume I, Editor Hill, N.

(5) Pierson, W., Editor Meteorological Papers, Vol. 2, No. 6, New York University, June 1960

(6) "Ocean Wave Spectra" Proceedings of a Conference, Prentice Hall, 1961

(7) Marsh, H. W., Schulkin, M., and Kneale, S. G., "Scattering of Underwater Sound by the Sea Surface", The Journal of the Acoustical Society of America, Vol. 33, No. 3.

(8) Nuttall, A., Cron, B. "Spectrum of a Signal Reflected from a Time-Varying Stochastic Surface" NUSC Research Report #NL-3013 September 1970.

(9) Cron, B., Streit, R., "Correlation of the Sea Surface for the Isotropic Case" USL Tech Memo #2211-221-69, July 29, 1969

(10) Cron, B., Hassell, B. and Keltonic, F. "Comparison of Theoretical and Experimental Values of Spatial Correlation", The Journal of the Acoustical Society of America, Vol. 37, No. 3, March 1965

(11) Axelrod, E., Schoomer, B., and Von Winkle, W., "Vertical Directionality of Ambient Noise in the Deep Ocean at a Site near Bermuda", The Journal of the Acoustical Society of America", Vol. 37, No. 1, January 1965

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